

A NOTE ON CONSISTENT ANOMALIES

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Abstract

Within a BRST formulation, we determine the expressions of the consistent anomaly for superstrings with extended worldsheet supersymmetries of rank N . We consider the $O(N)$ superconformal algebras up to $N = 4$, as well as the ‘small $N = 4$ ’ superalgebra. This is done using a superfield formalism, allowing to recover previous results that were expressed in components. Moreover, we identify the ‘small $N = 4$ ’ algebra as the constrained ‘large $N = 4$ ’ via a self-duality like condition in superspace.

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1 Introduction

In the last decades, after the work of Ademollo *et al.* [1], there has been a growing interest in string theories with extended worldsheet supersymmetries. More recently, various theories with different ranks N of local worldsheet supersymmetries have been shown to be related. Namely, there are more and more probes of an embedding of N -theories into $(N+1)$ -theories [2, 3, 4, 5]. Therefore, superstring theories with the higher rank N seem to be privileged. However, one crucial point in formulating such highly supersymmetric string theories is the delicate problem of the superconformal anomaly: its vanishing is one of the key ingredients to the consistency of a theory.

There are two different things in the vanishing of the anomaly. One thing is the cancelation of the coefficient related to the central charge of the superalgebra: it gives rise to the well-known critical dimension of the target space in which the string is embedded for $N \leq 2$. The second is the so-called consistent anomaly: it is responsible for the non-conservation of the symmetry generators, or, put differently, for the breaking of the Ward identities. Two of the authors showed in [6] that the coefficients of the superconformal anomalies vanish for all values of $N \geq 3$, but left quite unexplored the determination of the consistent anomalies. In this note, we fill this gap.

The equations that rule the gauge symmetries of superstrings with local extended worldsheet supersymmetry of arbitrary rank (N, N') were determined in [6]. The (super)-Beltrami parametrisation [7] being the most natural parametrisation allowing left/right factorization, it was generalized to the N -extended supersymmetric case, allowing to present results for the holomorphic sector only (it all trivially translates to the anti-holomorphic part).

Both classical and ghost conformally invariant multiplets are assembled into the components of a single $O(N)$ -superfield, which generalizes the Beltrami differential. This is briefly reviewed in the next section. In a path integral formulation, this ‘Beltrami-superfield’ would be the source for all linear superconformal generators, and indeed, the BRST symmetry that was obtained coincides with the one coming from usual operator product expansion treatments of the superconformal algebra. Since the BRST symmetry was found for the complete set of gauge fields and ghosts of conformal $2D$ -supergravity, we are in a position to investigate the consistent anomalies by means of the descent equations. It should be a local functional, with appropriate dimensions and ghost number, and has to satisfy a Wess and Zumino consistency equation [7].

In this note, we are using an $O(N)$ -superfield formalism [8] and we give in section 2 the

expressions of the superconformal consistent anomalies in terms of those superfields, for $N = 1, 2, 3$. These are in agreement with our former results [6], that were expressed in components.

Moreover, we have computed the anomalies in the yet unexplored cases of ‘small’ and ‘large’ $N = 4$ supersymmetry [1, 9]. We also indicate how the small $N = 4$ superconformal algebra can be deduced from the large $N = 4$, by some self-duality-like constraint imposed on the $O(N)$ -superfield. This constraint allows one to keep only half of the fields of the large $N = 4$ and is quite analogous to the reduction in [1] from 32 to 16 superconformal generators. This is presented in section 3, and compared with the results we obtain using operator product expansion techniques.

Finally, we present the intuitive fact that no consistent *local* expression of the anomaly can exist for $N > 4$. In fact, giving a sense to string theories with higher supersymmetries than $N = 4$ appears to be quite delicate: some generators of the superconformal algebra have negative weight and thus can not be physical.

2 Consistent anomaly for the $O(N)$ superconformal algebra

2.1 Conventions and notations

As described in [6], one can unify, in the Beltrami parametrisation, the $O(N)$ gauge superfield M^z and superghost C^z into a single superfield \hat{M}^z , graded with respect to the ghost number:

$$M^z = dz + \mu_{\bar{z}}^z d\bar{z} + \sum_{p=1}^N \frac{1}{p!} \theta^{i_1} \theta^{i_2} \dots \theta^{i_p} (m_{\bar{z}}^z)_{i_1 i_2 \dots i_p} d\bar{z}, \quad (1)$$

$$C^z = c^z + \sum_{p=1}^N \frac{1}{p!} \theta^{i_1} \theta^{i_2} \dots \theta^{i_p} c_{i_1 i_2 \dots i_p}^z, \quad (2)$$

$$\hat{M}^z = M^z + C^z. \quad (3)$$

The differential d and the BRST operator s are unified into a single graded operator \hat{d} such that the BRST equations for the gauge symmetry are expressed as a vanishing curvature condition in the N superspace

$$\begin{aligned} \hat{d} &= d + s, \\ \hat{d}\hat{M}^z &= \hat{M}^z \partial_z \hat{M}^z - \frac{1}{4} \sum_{i=1}^N (D_i \hat{M}^z)^2, \end{aligned} \quad (4)$$

with the superderivative $D_i = \partial_{\theta_i} + \theta_i \partial_z$ satisfying the anti-commutation relation

$$\{D_i, D_j\} = 2\delta_{ij} \partial_z, \quad (5)$$

required for the BRST algebra to close. Since we only deal with the holomorphic parts, we will not explicitly display the z and \bar{z} subscripts or indices anymore, keeping in mind the weights of the various fields and denoting ∂_z simply by ∂ .

The full BRST algebra can be obtained from the one for the superghosts [3, 10, 11], *i.e.*

$$sC = C\partial C - \frac{1}{4} \sum_{i=1}^N (D_i C)^2, \quad (6)$$

with the following correspondence

$$\begin{aligned} s &\rightarrow d + s, \\ C &\rightarrow M + C. \end{aligned} \quad (7)$$

Therefore, BRST transformations will be given for the ghosts (superfield or components), but the extension to the graded superfield (and therefore to the gauge superfield) is straightforward.

This same correspondence also enables one to simplify the problem of finding the consistent anomaly $\Delta_2^1(c_{i_1 \dots i_p}, m_{i_1 \dots i_p})$,^{*} as follows. After gauge fixing, the $m_{i_1 \dots i_p}$, $p = 0, \dots, N$ ($p = 0$ gives the Beltrami $\mu_{\bar{z}}^z$) become sources for the various currents in the functional integral. The anomalous Green functions for these currents are then obtained from suitable differentiation of the broken Ward identities for the effective action Γ (see [7]):

$$s(\Gamma) = \chi \int d^2x \Delta_2^1(m, c), \quad (8)$$

where the coefficient χ has to vanish for the superstring theory to be consistent. For $N \leq 2$, imposing $\chi = 0$ determines the dimension of the target space, whereas for any $N \geq 3$, the coefficient χ has been shown to be zero [6].

Since $s^2 = 0$, the consistent anomaly is constrained by the following Wess-Zumino consistency condition[†]

$$\int d^2x s\Delta_2^1 = 0 \quad \text{or} \quad s\Delta_2^1 = -d\Delta_1^2, \quad (9)$$

and the problem of finding the consistent anomaly thus reduces to a question of finding the cohomology of \hat{d} provided one defines a graded object $\hat{\Delta} = \Delta_0^3 + \Delta_1^2 + \Delta_2^1$ using the descent equations

$$\begin{aligned} s\Delta_2^1 + d\Delta_1^2 &= 0, \\ s\Delta_1^2 + d\Delta_0^3 &= 0, \\ s\Delta_0^3 &= 0. \end{aligned} \quad (10)$$

^{*}This is a 2-form with ghost number 1 according to the usual notation $X_{form \ degree}^{ghost \ number}$.

[†] Indeed, s and d respectively increase the ghost number and form degree by one.

Here, the correspondence (7) allows a great simplification: instead of the cohomology of \hat{d} , one can rather solve the cohomology of s at ghost number 3 (up to total derivatives since $\hat{d} = d + s$ and since a total derivative in Δ_0^3 always translates into a total derivative in Δ_2^1 and thus does not contribute). So the object we should look for is the 3-ghost 0-form $\Delta_0^3(c_{i_1 \dots i_p})$ that is BRST closed, up to total derivatives and s -exact terms (we will also refer to Δ_0^3 as to the consistent anomaly). The last step is the correspondence (7) that enables us to deduce the physical consistent anomaly Δ_2^1 .

2.2 Superfield formalism

The BRST algebra or the consistent anomaly can be computed within a component formalism, where one extracts the transformations for the various $c_{i_1 \dots i_p}$ ghosts as the $\theta^{i_1} \dots \theta^{i_p}$ terms. On the other hand, on supersymmetry grounds, the whole calculation can be done using a superfield formalism if one notices that $c_{i_1 \dots i_p}$ and its transformation are the lowest component of the superfields $D_{[i_1} \dots D_{i_p]} C$ and $s D_{[i_1} \dots D_{i_p]} C$ (where the brackets mean antisymmetrisation of the indices). The latter can be obtained from (6) using the fact that s and D_i anti-commute. This formalism exhibits the statistics and weights of the various ghosts, with C , D_i anticommuting and having respective weights -1 , $1/2$.

In the same way, we will obtain the consistent anomaly Δ_0^3 as the lowest component of an anomaly superfield in N -superspace, which is the unique completion of the $(N - 1)$ anomaly superfield by terms such that the whole superfield is s -closed, up to total derivatives and s -exact terms. The lowest component of this anomaly supermultiplet is the superconformal anomaly we are looking for, but the interpretation for its supersymmetric partners is still not clear.

In the case of $N = 0$ supersymmetry, the only 0-form with ghost number 3 which is s -closed (its transformation under (6) actually gives exactly zero and not a total derivative) is

$$\Delta_{N=0} = C \partial C \partial^2 C, \quad (11)$$

where C 's lowest component is related to the reparametrisation ghost c . Using the aforementioned correspondence (7), one can deduce the 1-ghost 2-form consistent anomaly in terms of the Beltrami differential and extract the expected $\partial_z^3 \mu_{\bar{z}}$ term, violating the conservation law of the energy momentum tensor.

Going up to $N = 1$ supersymmetry on the worldsheet, one now has a non-vanishing DC whose lowest component is the holomorphic part of the supersymmetric ghost, γ . Generalizing (11) to the supersymmetric case requires adding all possible 0-form, 3-ghost terms made out of

C and DC and selecting the BRST-closed combination. This leads to

$$\Delta_{N=1} = C \partial C \partial^2 C - C (\partial DC)^2 + \frac{1}{2} \partial C DC \partial DC, \quad (12)$$

whose s -transformation is again zero. The lowest component of (12) is in agreement with the results in [6] and the θ -term is found to be $s(\partial c \partial \gamma)$ so that it has no physical significance whatsoever.

The generalization to higher N worldsheet supersymmetries is identical: one considers all terms containing the non-vanishing $D_{[i_1} \cdots D_{i_p]} C$ for $p = 0, \dots, N$ and extracts the appropriate combination.

Here, the $O(N)$ structure becomes natural, and the $N = 4$ bound as to the existence of a consistent anomaly becomes manifest. Indeed, let us consider a generic term occurring in $\Delta_0^3(C)$. Since we are interested in terms that contribute to $\Delta_2^1(c, m)$, we should examine terms of the form[†]

$$C \partial^{a_1} D_{[i_1} \cdots D_{i_p]} C \partial^{a_2} D_{[i_1} \cdots D_{i_p]} C, \quad (13)$$

with $p \leq N$. To see what happens when N increases, we focus on the $p = N$ term. For this particular term, the total number of ∂_z derivatives $n = a_1 + a_2 = 3 - N$. Indeed, $n = 3 - p$ since the whole term has weight 0 and C, D_i have respective weights -1 and $\frac{1}{2}$. When we move from N to $N + 1$, $n \rightarrow n - 1$ until $n = 0$ for $N = 3$ so that one would not expect to increase N any further without encountering problems of locality.

However, moving to $N = 4$, one can notice that the ‘new’ superghost $D_1 D_2 D_3 D_4 C$ of weight $+1$ can be redefined as the derivative of a weight 0 ghost thanks to the fact that its BRST transformation is a total derivative. (This is due to the fact that the $O(4)$ superalgebra has a weight 0 generator which is related to the weight 1 generator of the $U(1)$ symmetry. This generator is to be associated with gauge field and ghost of respective weights $-1, 0$.)

The $N = 4$ anomaly can then be found as a local expression in terms of this weight 0 superghost that we will simply denote as $\partial^{-1} D_1 D_2 D_3 D_4 C$.

Defining the notation $\tilde{\Delta}_N$ for Δ_N with indices i, j, k, l running from 1 to $(N + 1)$ instead of N , the anomaly superfields can be expressed as follows:

$$\Delta_{N=2} = C \partial C \partial^2 C - C (\partial D_i C)^2 + \frac{1}{2} \partial C (D_i C) (\partial D_i C)$$

[†]In (13), the C is needed to extract a dz (we want to extract the $dz d\bar{z}$ term). The two other superghosts $D_{[i_1} \cdots D_{i_p]} C$ are the same, because starting from $\Delta_2^1(c, m)$, one has to differentiate with respect to $c_{i_1 \dots i_p}$ and $m_{i_1 \dots i_p}$ to obtain the anomalous conservation law for the corresponding generator.

$$+ \frac{1}{2} C(D_{[i}D_j]C)(\partial D_{[i}D_j]C) + \frac{1}{2} D_i C(\partial D_j C)(D_{[i}D_j]C), \quad (14)$$

$$\begin{aligned} \Delta_{N=3} &= \tilde{\Delta}_{N=2} \\ &- C(D_{[i}D_jD_k]C)^2 + \frac{1}{2} (D^{[i}D_jC)(D^jD_kC)(D^{k]}D_iC) \\ &+ \frac{1}{2} (D_{[i}D_jD_k]C)(D_{[i}D_j]C)(D_kC), \end{aligned} \quad (15)$$

$$\begin{aligned} \Delta_{N=4} &= \tilde{\Delta}_{N=3} \\ &+ \left[\frac{1}{2} (D_{[i}D_jD_k]C)(D_lC) - C(D_{[i}D_jD_kD_l]C) \right] (\partial^{-1} D_{[i}D_jD_kD_l]C). \end{aligned} \quad (16)$$

We have checked by inspection, using weight and ghost number constraints, that these expressions of the anomalies are the most general ones (up to s -exact terms that we do not mention since they have no physical significance). For $N \leq 3$, the s -transformation of the anomaly superfields is 0, but $\Delta_{N=4}$ transforms into a total derivative. The additional part for $N = 4$ happens to be the aforementioned weight 0 superghost times its BRST transformation:

$$\Delta_{N=4} = \tilde{\Delta}_{N=3} - s(\partial^{-1} D_{[i}D_jD_kD_l]C) (\partial^{-1} D_{[i}D_jD_kD_l]C). \quad (17)$$

All these results as well as the BRST transformations can be re-expressed in components for the ghosts or gauge fields. The lowest components of the former anomaly superfields give the consistent anomalies in terms of ghosts that were found in [6] for $N \leq 3$ (eq. (39) therein).

In the following, we will present our results in components for the $N = 4$ case. To obtain the physical consistent anomaly Δ_2^1 , one should first use the correspondence (7) to write $\Delta_0^3(M^z + C^z)$ from $\Delta_0^3(C^z)$, and then extract the $dzd\bar{z}$ term with a single ghost. This is done explicitly for $N = 4$ in the next subsection.

2.3 Component formalism for the $N = 4$ case

The gauge fields and ghosts will be denoted as

fields	μ	α	ρ	φ	β	c	γ	ϱ	δ	b
statistics	+	−	+	−	+	−	+	−	+	−
weight	0	$+\frac{1}{2}$	+1	$+\frac{3}{2}$	+1	−1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	0

with the following graded fields:

$$\hat{\mu} = dz + \mu d\bar{z} + c,$$

$$\begin{aligned}
\hat{\alpha}_i &= \alpha_i d\bar{z} + \gamma_i, \\
\hat{\rho}_{ij} &= \rho_{ij} d\bar{z} + \varrho_{ij}, \\
\hat{\varphi}_i &= \varphi_i d\bar{z} + \delta_i, \\
\hat{\beta} &= \beta d\bar{z} + b,
\end{aligned} \tag{18}$$

where we use the duals $\tilde{c}_{i_{p+1}\dots i_N}$ rather than the fields $c_{i_1\dots i_p}$ themselves if $p \geq N/2$.[§]

Namely our ghost superfield for the large $N = 4$ reads

$$C = c + \theta^i \gamma_i + \frac{1}{2} \theta^i \theta^j \epsilon_{ijkl} \varrho_{kl} + \frac{1}{3!} \theta^i \theta^j \theta^k \epsilon_{ijkl} \delta_l + \frac{1}{4!} \theta^i \theta^j \theta^k \theta^l \epsilon_{ijkl} \partial b. \tag{19}$$

Note that the last term in the superghost C is written as the derivative of the $U(1)$ ghost b (weight 0). This is how the problem of non-locality is avoided, thanks to the fact that the ‘dimension 1 ghost’ can be taken to be ∂b .

The BRST transformations for the ghosts derived from (6) are

$$\begin{aligned}
s c &= c \partial c - \frac{1}{4} \gamma_i^2, \\
s \gamma_i &= c \partial \gamma_i - \frac{1}{2} \gamma_i \partial c - \frac{1}{2} \epsilon_{ijkl} \gamma_j \varrho_{kl}, \\
s \varrho_{ij} &= c \partial \varrho_{ij} + \frac{1}{4} \epsilon_{ijkl} \gamma_k \partial \gamma_l - \frac{1}{4} (\delta_i \gamma_j - \delta_j \gamma_i) - \frac{1}{2} \epsilon_{ijkl} \varrho_{km} \varrho_{ml}, \\
s \delta_i &= c \partial \delta_i + \frac{1}{2} \delta_i \partial c - \partial \varrho_{ij} \gamma_j - \frac{1}{2} \partial b \gamma_i - \frac{1}{2} \epsilon_{ijkl} \delta_j \varrho_{kl}, \\
s b &= c \partial b - \frac{1}{2} \gamma_i \delta_i.
\end{aligned} \tag{20}$$

The consistent anomaly for the ghosts is obtained as the lowest component of (16) and the correspondence (7) is used to obtain its expression for the graded fields:

$$\begin{aligned}
\hat{\Delta} &= \hat{\mu} \partial \hat{\mu} \partial^2 \hat{\mu} - \hat{\mu} (\partial \hat{\alpha}_i)^2 + \frac{1}{2} \partial \hat{\mu} \hat{\alpha}_i \partial \hat{\alpha}_i + 2 \hat{\mu} \hat{\rho}_{ij} \partial \hat{\rho}_{ij} - \hat{\mu} (\hat{\varphi}_i)^2 \\
&- \frac{1}{2} \epsilon_{ijkl} \hat{\alpha}_i \partial \hat{\alpha}_j \hat{\rho}_{kl} + \frac{1}{3} \epsilon_{ijkl} \hat{\rho}_{ij} \hat{\rho}_{km} \hat{\rho}_{ml} - \hat{\rho}_{ij} \hat{\alpha}_i \hat{\varphi}_j + \frac{1}{2} \hat{\alpha}_i \hat{\varphi}_i \hat{\beta} + \hat{\mu} \hat{\beta} \partial \hat{\beta}.
\end{aligned} \tag{21}$$

One then extracts the $dzd\bar{z}$ terms with one ghost to obtain the physical anomaly occurring in the broken Ward identities as

$$\Delta_2^1 = 2 (c \partial_z^3 \mu - \gamma_i \partial_z^2 \alpha_i - 2 \varrho_{ij} \partial_z \rho_{ij} + \delta_i \varphi_i - b \partial \beta). \tag{22}$$

[§]We define the duals by $c_{i_1\dots i_p} = \epsilon_{i_1\dots i_N} \tilde{c}_{i_{p+1}\dots i_N}$. Our normalization for the totally antisymmetric tensor is $\epsilon_{12\dots N} = 1$.

3 ‘Duality’ and the small $N = 4$

3.1 Duality-like conditions towards the small $N = 4$

It is well known that the $O(4)$ superconformal algebra (of central charge 0) has two independent $SU(2)$ subalgebras, of canceling central charges. These are the so-called ‘small $N = 4$ ’ superconformal algebras and are obtained by proper truncation of the ‘large $N = 4$ ’. In our formalism, the reduction to the small $N = 4$ is done by imposing (anti)self-duality-like conditions on the superfields. It can be viewed in two different ways: in component formalism, the single superfield condition

$$\left(D_{[i} D_{j]} + \frac{1}{2} \epsilon_{ijkl} D_k D_l \right) C = 0, \quad (23)$$

(or equivalently with a $-$ sign) is compatible with the BRST algebra and allows one to reduce the independent component ghosts to c , γ_i and $c_{ij}^+ = \frac{1}{2} (\varrho_{ij} + \frac{1}{2} \epsilon_{ijkl} \varrho_{kl})$. Indeed, (23) written in components gives the relations

$$b = \partial c, \quad (24)$$

$$\delta_i = \partial \gamma_i, \quad (25)$$

$$\varrho_{ij} = \frac{1}{2} \epsilon_{ijkl} \varrho_{kl}, \quad (26)$$

The latter are obtained by examining not only the lowest component condition, but all of them, that is all the different $\theta^{i_1} \dots \theta^{i_p}$ terms. These identifications of all components enables us to rewrite the ‘dualised’ anomaly as the one for the small $N = 4$ derived below simply by ${}^+ \Delta = 2\Delta_{small}$ where

$${}^+ \Delta_0^3 = 2 [c \partial c \partial^2 c - c (\partial \gamma)^2 - \frac{1}{4} \partial^2 c \gamma^2 + c \varrho_{ij} \partial \varrho_{ij} - \gamma_i \partial \gamma_j \varrho_{ij} + \frac{1}{3} \varrho_{ij} \varrho_{ik} \varrho_{kj}], \quad (27)$$

$${}^+ \Delta_2^1 = 4 [c \partial^3 \mu - \gamma_i \partial^2 \alpha_i - \varrho_{ij} \partial \rho_{ij}]. \quad (28)$$

On the other hand, if one wants to remain in the $O(N)$ -superfield formalism and ‘dualise’ the anomaly superfield (16), then in addition to (23), two other duality-like conditions are to be imposed, reproducing (24, 25) when reduced to their lowest components.

3.2 Small $N = 4$ superconformal algebra and consistent anomaly

We have also derived the consistent anomaly for the small $N = 4$ superconformal algebra, using usual OPE techniques. Here we present our results and provide a check of the duality-like constraint, by identifying the two small $N = 4$ anomalies.

The small $N = 4$ superconformal algebra consists of stress-energy tensor T , four supercurrents G_a^\pm ($a = 1, 2$), and $SU(2)$ currents J_i ($i = 1, 2, 3$). Their respective weights are 2, $\frac{3}{2}$, 1, and their operator product expansions can be found, for example, in [1, 12]. Introducing the corresponding ghosts $c, \gamma_{\pm a}, c_i$ (weights $-1, -\frac{1}{2}, 0$) and antighosts $b, \beta_{\pm a}, b_i$, their operator product expansions are

$$\begin{aligned} c(z)b(w) &\sim \frac{1}{z-w}, \\ \gamma_{\pm a}(z)\beta_{\mp b}(w) &\sim \frac{\delta_{ab}}{z-w}, \\ c_i(z)b_j(w) &\sim \frac{\delta_{ij}}{z-w}. \end{aligned} \tag{29}$$

One then computes the BRST charge for this system and asks for its nilpotency, which requires the central charge of the ghost system to be $c = +12$ so that the matter part must have negative central charge -12 (this corresponds to the well-know critical ‘dimension’ -2 for the target space). The BRST charge allows to find the transformations for the ghosts by usual OPE calculation as

$$\begin{aligned} sc &= c\partial c - \gamma_{+1}\gamma_{-1} - \gamma_{+2}\gamma_{-2}, \\ s\gamma_{+1} &= c\partial\gamma_{+1} - \frac{1}{2}\gamma_{+1}c_3 - \frac{1}{2}\gamma_{+1}\partial c - \frac{1}{2}c_1\gamma_{+2} + \frac{i}{2}c_2\gamma_{+2}, \\ s\gamma_{+2} &= c\partial\gamma_{+2} - \frac{1}{2}\gamma_{+1}c_1 - \frac{1}{2}\gamma_{+2}\partial c + \frac{1}{2}c_3\gamma_{+2} - \frac{i}{2}c_2\gamma_{+1}, \\ s\gamma_{-1} &= c\partial\gamma_{-1} + \frac{1}{2}\gamma_{-1}c_3 - \frac{1}{2}\gamma_{-1}\partial c + \frac{1}{2}c_1\gamma_{-2} + \frac{i}{2}c_2\gamma_{-2}, \\ s\gamma_{-2} &= c\partial\gamma_{-2} + \frac{1}{2}\gamma_{-1}c_1 - \frac{1}{2}\gamma_{-2}\partial c - \frac{1}{2}c_3\gamma_{-2} - \frac{i}{2}c_2\gamma_{-1}, \\ sc_1 &= c\partial c_1 - ic_2c_3 + \gamma_{+1}\partial\gamma_{-2} + \gamma_{+2}\partial\gamma_{-1} - \partial\gamma_{+1}\gamma_{-2} - \partial\gamma_{+2}\gamma_{-1}, \\ sc_2 &= c\partial c_2 + ic_1c_3 + i\gamma_{+1}\partial\gamma_{-2} - i\gamma_{+2}\partial\gamma_{-1} - i\partial\gamma_{+1}\gamma_{-2} + i\partial\gamma_{+2}\gamma_{-1}, \\ sc_3 &= c\partial c_3 - ic_1c_2 + \gamma_{+1}\partial\gamma_{-1} - \gamma_{+2}\partial\gamma_{-2} - \partial\gamma_{+1}\gamma_{-1} + \partial\gamma_{+2}\gamma_{-2}. \end{aligned} \tag{30}$$

The consistent anomaly, as the BRST invariant zero-form with ghost number 3, is then found to be

$$\begin{aligned} \Delta_{small} &= c\partial c\partial^2 c - c c_i \partial c_i - i c_1 c_2 c_3 \\ &\quad - 4c(\partial\gamma_{+1}\partial\gamma_{-1} + \partial\gamma_{+2}\partial\gamma_{-2}) + \partial c(\gamma_{+1}\gamma_{-1} + \gamma_{+2}\gamma_{-2}) \\ &\quad + c_1(\gamma_{+1}\partial\gamma_{-2} - \gamma_{-1}\partial\gamma_{+2} + \gamma_{+2}\partial\gamma_{-1} - \gamma_{-2}\partial\gamma_{+1}) \\ &\quad + i c_2(\gamma_{+1}\partial\gamma_{-2} + \gamma_{-1}\partial\gamma_{+2} - \gamma_{+2}\partial\gamma_{-1} - \gamma_{-2}\partial\gamma_{+1}) \\ &\quad + c_3(\gamma_{+1}\partial\gamma_{-1} - \gamma_{-1}\partial\gamma_{+1} - \gamma_{+2}\partial\gamma_{-2} + \gamma_{-2}\partial\gamma_{+2}). \end{aligned} \tag{31}$$

This anomaly can be re-expressed in a more concise form, using the vector $\Gamma_{\pm} \equiv (\gamma_{\pm 1}, \gamma_{\pm 2})$ and neglecting total derivatives, as

$$\begin{aligned} \Delta_{small} = & \ c \partial c \partial^2 c - c c_i \partial c_i - c \left(\partial^2 \Gamma_+^t \Gamma_- + 6 \partial \Gamma_+^t \partial \Gamma_- + \Gamma_+^t \partial^2 \Gamma_- \right) \\ & - \partial \Gamma_+^t c_i \bar{\sigma}^i \Gamma_- + \Gamma_+^t c_i \bar{\sigma}^i \partial \Gamma_- - i c_1 c_2 c_3, \end{aligned} \quad (32)$$

where the superscript t means transpose of the vector Γ_+ .

The anomaly expressed as (31) is to be identified with the ‘dualised’ anomaly $(^+ \Delta)/2$ of subsection 3.1, provided the different ghosts are related by

$$\begin{aligned} \gamma_{\pm 1} &= \frac{1}{2} (\gamma_1 \pm i \gamma_2), \quad \gamma_{\pm 2} = \frac{1}{2} (\gamma_3 \pm i \gamma_4), \\ c_1 &= 2i \varrho_{14}, \quad c_2 = -2i \varrho_{13}, \quad c_3 = 2i \varrho_{12}. \end{aligned} \quad (33)$$

4 Concluding remarks

According to the weight counting discussed below eq. (13), one does not expect to be able to increase N further in order to obtain local consistent anomalies. When applying the mechanism depicted above to compute a consistent anomaly for the $N = 5$ case, one encounters serious problems of locality: there is no way to construct a *local* anomaly because the ‘overweighted’ ghosts (weights ≥ 1) do not transform into total derivatives anymore. Thus one would be forced to consider terms with two negative powers of the ∂_z derivative, which can hardly be given a sense.

This can be understood from previous knowledge. Indeed, the superconformal anomaly comes from the central term of the algebra, and K. Schoutens showed in [8] that it is not possible to add a central extension to an $O(N)$ superconformal algebra for $N \geq 5$. Moreover, such an algebra would have generators of negative weights (to which our overweighted ghosts are associated) so that there seems to be no way to consistently define string theories with $O(N)$ -extended worldsheet supersymmetry for $N > 4$ unless the negative weighted generators can be made unphysical.

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